ANNUAL JOURNAL OF ELECTRONICS, 2014, ISSN 1314-0078

Analysis of a Synchronous DC-DC Converter Working between two Voltage Sources

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Abstract – **A synchronous dc-dc converter is analyzed, working with voltage sources connected to its input and output, with applications for the power train of an electric vehicle with hybrid energy source (supercapacitor and a battery). The obtained results are plotted and adequate approximations are made allowing fast hand calculations during the design phase. Key practical considerations are also mentioned.**

Keywords – **Bidirectional DC/DC converter, Electric Vehicle, Synchronous buck converter**

I. INTRODUCTION

As a recent publication indicated, an upcoming 67% increment in the production of electric and hybrid electric vehicles in the upcoming year, will result in increased interest for their power train architectures and power efficiency [1]. The power electronic converters used in their design are the main topic of this paper.

One of the main methods to increase the power efficiency of the system is to "close the loop" around the energy conversion process. The main idea is to keep the energy supplied from the onboard energy source in mechanical or electrical form and to transform it back and forth depending on the need. During acceleration the electric motor uses the energy supplied thought a power electronic converter from the energy source to produce torque and drive the vehicle. To close the loop during braking the power electronic converters onboard must be able to supply back the energy from the electric generator to the power source, which usually is a battery.

However, the main use of electric vehicle in urban environments involving rapid acceleration, and braking uses the battery in a rather suboptimal way, in view of the non-flat power load profile. To obtain a more balanced profile, a secondary onboard energy source is installed, supplying the peak power requirements during motion, leaving the battery to be chosen based only on mean energy balance. The most popular choices for the secondary power source are either a supercapacitor, or a flywheel [2],[3].

Although there is a great variety of power train architecture with differences concerning the amount of power electronic converter, and their connections with the onboard power sources [4], the main focus here is a cascade connection of the battery, supercapacitor and motor and their power converters, as shown in Fig. 1. The analysis of the possible modes of operation for the bidirectional dc-dc converter between the supercapacitor and the battery, working with voltage sources in its input and output, and the comparison with the standard converter working with passive load is the motivation for this paper.

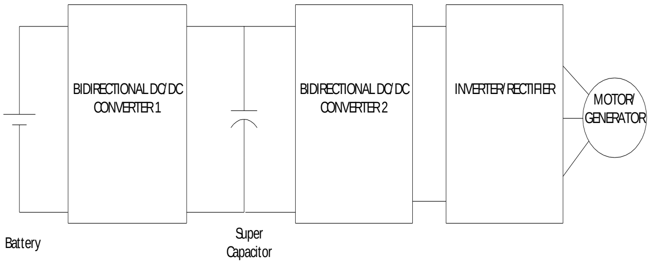

Fig. 1. Cascade power train architecture for electric vehicles

The paper is structured as follows: in section II the bidirectional dc-dc converter is analyzed and expressions are obtained for the maximum, average and ripple current. These expressions are approximated for the common case of circuit parameters allowing simplified expressions to be used in the early design phase. In section III, the obtained expressions are plotted as a family of curves, for different circuit parameters allowing additional conclusions to be made about the optimal steady state regime. These are gathered in section IV along with additional comments for potential practical pitfalls.

II. ANALYSIS

The analyzed dc-dc converter connected between the battery and the supercapacitor in the power train architecture is shown in Fig. 2, along with its equivalent circuits during the two possible operation modes.

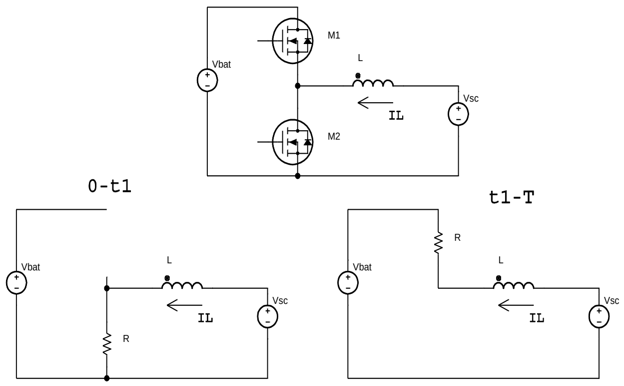

Fig. 2. Analyzed power converter and its equivalent circuits

The large capacitance value of the supercapacitor prevents large variations to the voltage across it, if the analysis is carried over short time durations, allowing to be modelled with a voltage source for the purposes of

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analysis. Additionally the following approximations are made during the analysis:

- the switching dynamics of the power transistors are ignored;
- the MOS transistors are modeled with linear time-invariant resistance R_1 in their linear mode, and assumed not conducting in reverse direction;
- the battery and the supercapacitor are modeled as an ideal voltage source in series with linear equivalent series resistance;
- the inductor is modeled with linear time invariant series combination of inductance L and resistance R_2.

Based on the above assumptions the following differential equations can be written for the two equivalent circuits, where $\tau = \frac{L}{R}$, $R = R_1 + R_2$:

$$\frac{di_L(t)}{dt} + \frac{i_L(t)}{\tau} = \frac{V_{sc}}{L} \qquad t \in (0, t_1)$$
$$\frac{di_L(t)}{dt} + \frac{i_L(t)}{\tau} = \frac{V_{sc} - V_{bat}}{L} \qquad t \in (t_1, T) \qquad (1)$$

The above equations are put in nondimensional form by a change of variables using the time period T, where *t=s.T*:

$$\frac{1}{\mathrm{T}}\frac{di_L(s)}{ds} + \frac{i_L(s)}{\tau} = \frac{V_{sc}}{L} \qquad s \in \left(0, \frac{t_1}{\mathrm{T}}\right)$$
$$\frac{1}{\mathrm{T}}\frac{di_L(s)}{ds} + \frac{i_L(s)}{\tau} = \frac{V_{sc} - V_{bat}}{L} \qquad s \in \left(\frac{t_1}{\mathrm{T}}, 1\right) \qquad (2)$$

The next step is to solve the equations using the intinal conditions $i_L(0) = I_{L_0}, i_L(\frac{t_1}{T}) = I_{L_1}$:

-for $s \in \left(0, \frac{t_1}{T}\right)$

$$i_L(s) = \frac{V_{sc}}{R}\left(1 - \exp(-\frac{Ts}{\tau})\right) + I_{L_0}\exp\left(-\frac{Ts}{\tau}\right) \quad (3)$$

-for $s \in \left(\frac{t_1}{T}, 1\right)$

$$i_L(s) = \frac{V_{sc} - V_{bat}}{R}\left(1 - \exp(-\frac{Ts}{\tau})\exp(\frac{t_1}{\tau})\right) + I_{L_1}\exp(-\frac{Ts}{\tau})\exp(\frac{t_1}{\tau}) \qquad (4)$$

Solving (3) for $\mathrm{s} = \frac{t_1}{T}$ a expression is obtained for the initial contidtion of the second equation:

$$i_L\left(\frac{t_1}{T}\right) = \frac{V_{sc}}{R}\left(1 - \exp(-\frac{t_1}{\tau})\right) + I_{L_0}\exp(-\frac{t_1}{\tau}) \quad (5)$$

Assuming also steady- state operation of the converter, so that $i_L(1) = i_L(0)$, the second equation can be written as:

$$i_L(1) = \frac{V_{sc} - V_{bat}}{R}\left(1 - \exp(-\frac{T}{\tau})\exp(\frac{t_1}{\tau})\right) + I_{L_1}\exp\left(-\frac{T}{\tau}\right)\exp(\frac{t_1}{\tau}) \quad (6)$$

Substituting (5) in (6) and using the steady-state assumption the following is obtained:

$$I_L(0) = \frac{V_{sc} - V_{bat}}{R}\left(1 - \exp(-\frac{T}{\tau})\exp(\frac{t_1}{\tau})\right) + \left(\frac{V_{sc}}{R}\left(1 - \exp(-\frac{t_1}{\tau})\right) + I_{L_0}\exp(-\frac{t_1}{\tau})\right)\exp(-\frac{T}{\tau})\exp(\frac{t_1}{\tau}) \quad (7)$$

This allows an expression to be obtained for the maximum and minumum current:

$$I_L(0) = I_{min} = \frac{V_{sc}}{R} - \frac{V_{bat}}{R}\left(\frac{1 - \exp(-\frac{T}{\tau})\exp(\frac{t_1}{\tau})}{1 - \exp(-\frac{T}{\tau})}\right)$$

$$I_L\left(\frac{t_1}{T}\right) = I_{max} = \frac{V_{sc}}{R} - \frac{V_{bat}}{R}\left(\frac{\exp(-\frac{t_1}{\tau}) - \exp(-\frac{T}{\tau})}{1 - \exp(-\frac{T}{\tau})}\right) (8)$$

Depending on the relative magnitudes of the two currents three possible modes of operation are possible:

- $I_{min} > 0, I_{max} > 0$ The current flows only through T2 and D1- this is known as first quadrant operation , and the energy flows from the supercapacitor to the battery;
- $I_{min} < 0, I_{max} < 0$ The current flows through T1 and D2 – the converter works only in second quadrant and the energy flows from the battery to the supercapacitor;
- $I_{min} < 0, I_{max} > 0$ As a combination of the above two the current flows through T1,T2 , D1 and D2, and the converter work in both first and second quadrant. The energy direction in this mode is obtained based on the average value of the inductor current.

Average Current

An expression for the average current is obtained by integrating the inductor current for the two time periods:

-For $s \in \left(0, \frac{t_1}{T}\right)$:

$$I_{L_{av}} = \frac{T}{t_1}\int_0^{\frac{t_1}{T}}\left(\frac{V_{sc}}{R}\left(1 - \exp(-\frac{Ts}{\tau})\right) + I_{L_0}\exp(-\frac{Ts}{\tau})\right)ds =$$
$$= \frac{V_{sc}}{R}\left(1 + \frac{\tau}{t_1}\left(\exp(-\frac{t_1}{\tau}) - 1\right)\right) - I_L(0)\frac{\tau}{t_1}\left(\exp(-\frac{t_1}{\tau}) - 1\right) \quad (9)$$

- For $s \in \left(\frac{t_1}{T}, 1\right)$

$$I_{L_{av}} = \frac{T}{T - t_1}\int_{\frac{t_1}{T}}^{1}\left(\frac{V_{sc} - V_{bat}}{R}\left(1 - \exp(-\frac{Ts}{\tau})\exp(\frac{t_1}{\tau})\right) + I_{L_1}\exp(-\frac{Ts}{\tau})\exp(\frac{t_1}{\tau})\right)ds =$$
$$= \frac{V_{sc} - V_{bat}}{R}\left(1 + \frac{\tau}{T - t_1}\left(\exp\left(-\frac{T}{\tau}\right)\exp(\frac{t_1}{\tau}) - 1\right)\right) - I_{L_1}\frac{\tau}{T - t_1}\left(\exp\left(-\frac{T}{\tau}\right)\exp(\frac{t_1}{\tau}) - 1\right) \quad (10)$$

Summing (9) and (10) the average current is:

$$I_{L_{av}} = I_{L_{av}}\Big|_0^{\frac{t_1}{T}} + I_{L_{av}}\Big|_{\frac{t_1}{T}}^{1} = \frac{V_{sc}}{R} + \frac{V_{bat}\tau}{Rt_1}\left(\exp(-\frac{t_1}{\tau}) - 1\right)\frac{1 - \exp(-\frac{T}{\tau})\exp(\frac{t_1}{\tau})}{1 - \exp(-\frac{T}{\tau})}$$
$$\frac{V_{sc} - V_{bat}}{R} + \frac{V_{bat}}{R}\left(\frac{\tau}{T - t_1}\left(\exp\left(-\frac{T}{\tau}\right)\exp(\frac{t_1}{\tau}) - 1\right)\right)\left(\frac{\exp(-\frac{-t_1}{\tau}) - \exp(-\frac{T}{\tau})}{1 - \exp(-\frac{T}{\tau})} - 1\right)$$

$$= 2\frac{V_{sc}}{R} + \frac{V_{bat}}{R}\left(\frac{\exp(-\frac{t_1}{\tau}) - 1}{1 - \exp(-\frac{T}{\tau})}\right)\left(1 - \exp\left(-\frac{T}{\tau}\right) * \exp(\frac{t_1}{\tau})\right)\left(\frac{\tau}{t_1} - \frac{\tau}{T - t_1}\right) - \frac{V_{bat}}{R} \quad (11)$$

Based on the above expression, and the assumed direction of the inductor current two situations can arise:

- $I_{Lav} > 0$ The energy is flowing from the supercapacitor to the battery;
- $I_{Lav} < 0$ Energy is flowing from the battery to the supercapacitor.

Maximum and ripple current

Putting $\frac{t_1}{T} = D$ the following expressions are obtained for the maximum and minimum current :

$$I_{L_{min}} = \frac{\left(2\frac{V_{sc}}{R} - \frac{V_{bat}}{R} - I_{Lav}\right)}{\left(\exp(\frac{-t_1}{\tau}) - 1\right)\frac{\tau}{T}\left(\frac{1}{D} - \frac{1}{1-D}\right)} + \frac{V_{sc}}{R} \quad (12)$$

$$I_{L_{max}} = \frac{\left(2\frac{V_{sc}}{R} - \frac{V_{bat}}{R} - I_{Lav}\right)\exp(\frac{-t_1}{\tau})}{\left(\exp(\frac{-t_1}{\tau}) - 1\right)\frac{\tau}{T}\left(\frac{1}{D} - \frac{1}{1-D}\right)} + \frac{V_{sc}}{R} \quad (13)$$

Using the relation $\Delta I_L = I_{L_{max}} - I_{L_{min}}$, a expression for the ripple current is obtained:

$$\Delta I_L = \frac{\left(2\frac{V_{sc}}{R} - \frac{V_{bat}}{R} - I_{Lav}\right)}{\left(\exp(\frac{-t_1}{\tau}) - 1\right)\frac{\tau}{T}\left(\frac{1}{D} - \frac{1}{1-D}\right)}\left(\exp(\frac{-t_1}{\tau}) - 1\right) = \frac{\left(2\frac{V_{sc}}{R} - \frac{V_{bat}}{R} - I_{Lav}\right)}{\frac{\tau}{T}\left(\frac{1}{D} - \frac{1}{1-D}\right)}$$

$$\Delta I_L = \frac{-V_{bat}}{R}\frac{\exp(\frac{-t_1}{\tau}) - 1}{1 - \exp(\frac{-T}{\tau})}\left(1 - \exp(\frac{-T}{\tau})\exp(\frac{t_1}{\tau})\right) \quad (14)$$

Approximate expressions

To obtain approximate forms of the above nonlinear equations they are expressed in Taylor series neglecting higher terms. The condition, allowing the approximate analysis is $\frac{T}{\tau} << 1$. This surmounts the fact that the circuit time constant is very large in comparison to the period generated by the control circuit.

For the ripple current the following expression is obtained:

$$\Delta I_L \approx \frac{-V_{bat}}{R}\frac{\left(1 - \frac{t_1}{\tau}\right) - 1}{1 - \left(1 - \frac{T}{\tau}\right)}\left(1 - \left(1 + \frac{t_1}{\tau}\right)\left(1 - \frac{T}{\tau}\right)\right) =$$

$$= \frac{V_{bat}}{R}\frac{t_1}{T}\left(1 - \left(1 + \frac{t_1 - T}{\tau} - \frac{t_1 T}{\tau^2}\right)\right) \approx \frac{V_{bat}}{R}D\left(\frac{1 - D}{\frac{\tau}{T}}\right)$$

$$= \frac{V_{bat}}{L}DT(1 - D) \quad (15)$$

The current ripple is a design parameter based on the optimal charging of the battery without causing excessive overheating. The maximum value is at $D = \frac{1}{2}$, and choosing the working frequency, an inital value of the inductance can be calculated.

III. SIMULATION

The following parameters are used for the plotting of the various circuit waveforms: Vsc=48[V], Vbat=55[V], R=0.1[Ohm], tau=0.01[s], f=20[kHz].

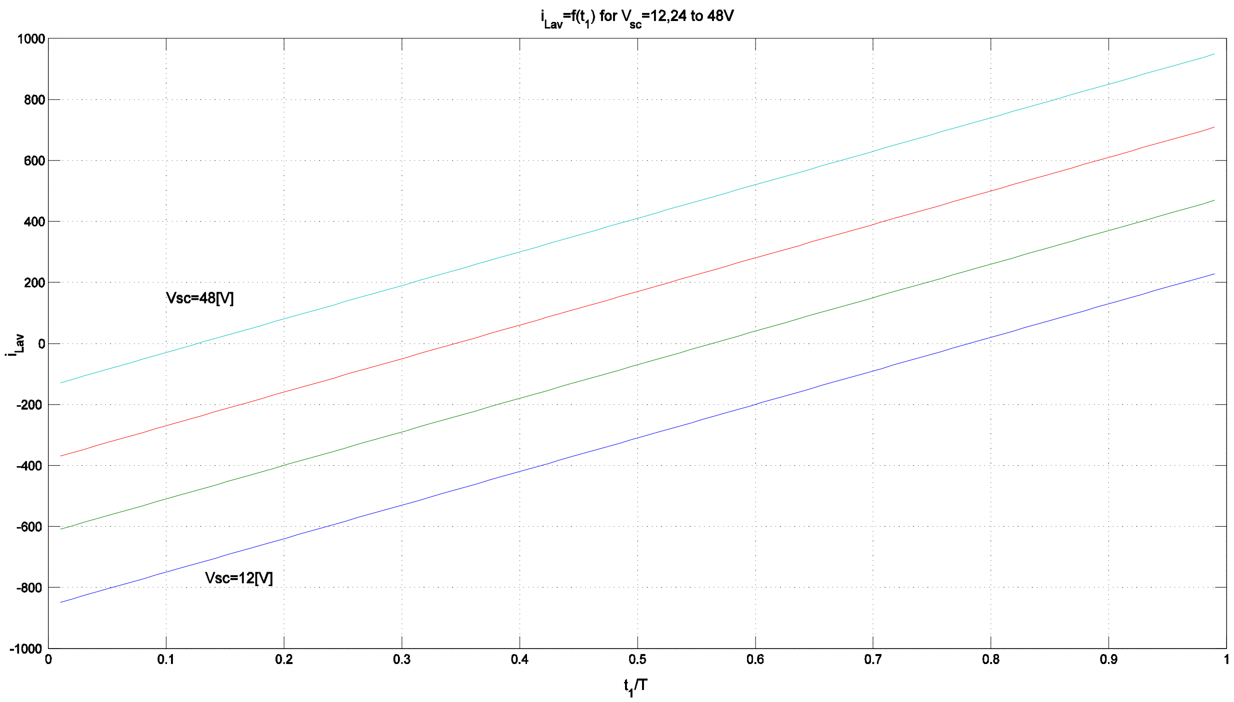

Fig. 3. Average inductor current as a function of the duty cycle with parameter the supercapacitor voltage

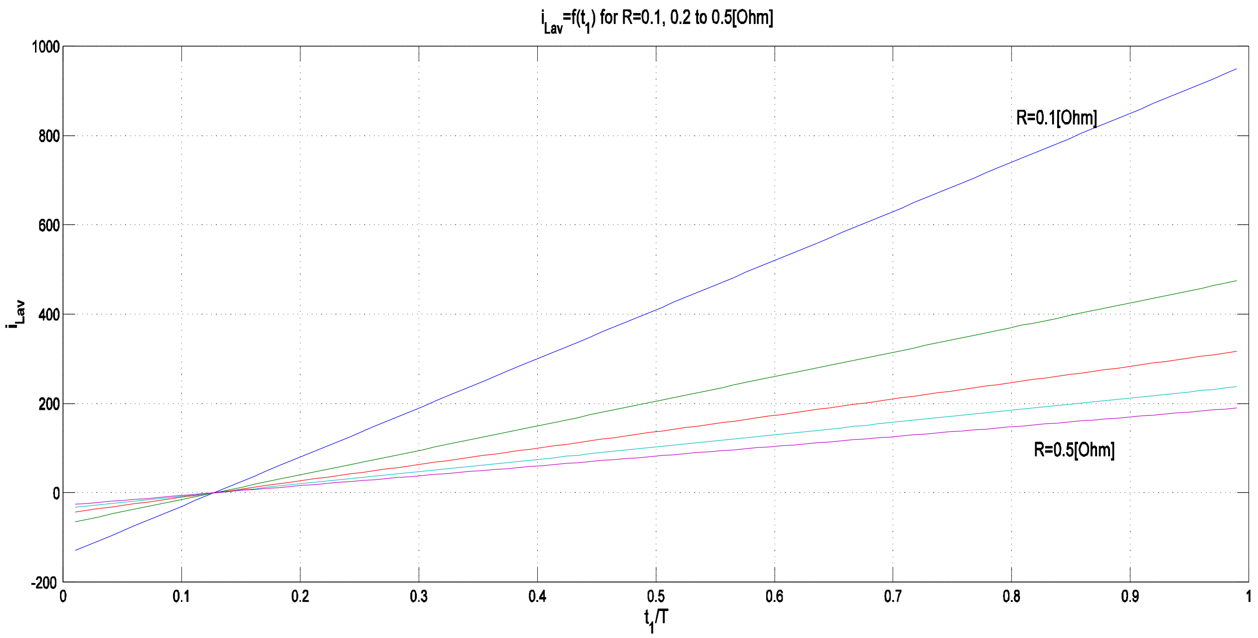

Fig. 4. Average inductor current as a function of the duty cycle with parameter the circuit resistance

The average inductor current, as a function of the duty cycle, with parameter the supercapacitor voltage is shown in Fig. 3. If the circuit resistance is chosen as the parameter the results are shown in Fig. 4.

IV. CONCLUSION

The paper analyzed a synchronous dc-dc converter connected between two voltage sources and expressions were obtained for the circuit variables.

A key difference in the circuit operation, in contrast to the typical passive load, is that the discontinuous conduction mode, characterized with the inductor current reaching zero before the end of the cycle, is not present. When the inductor current reaches zero it just reverses direction. This can lead to some practical considerations when designing the control circuit with typical integrated circuits generating the synchronous impulses for the two transistors containing soft start functionality. During soft start typically the duty cycle for one of the transistors, for example M1, slowly increases from zero to the steady-state value controlled by the closed loop, and as M2 is controlled synchronously, its duty cycle decreases from maximum toward the steady state value. In this case the rate at which the current rises is limited only by the inductor and its magnitude, only by the small active resistance in the circuit. The inductance value is chosen, as noted, based on steady state ripple considerations and this can lead to substantial current overloading during transient operation. If saturation effects are included the situation can be additionally aggravated. The possible solutions to the described problem are either changing the control algorithm not to use synchronous control with soft start, or substitute M2 with a diode. The second solution is inferior, because the resulting circuit is not able to transfer energy in both directions, as it is able only of a one quadrant operation. Additionally, using a diode instead of reverse conducting MOS transistor increases conduction losses for high current levels.

The effects of steady-state regulator errors can also be studied based on the conducted mathematical analysis. This error leads to duty cycle variation around the ideal value. As can be seen from figure 4, this variation can lead to serious transistor current overloading, if the resistance is small. The inductance value has very little influence over the average current, if the approximate expression $\frac{T}{\tau} << 1$ is satisfied. For more detailed analysis the full equation should be used.

Finally, although the paper concerned itself only with electric vehicle applications, the analyzed converter can be used as an intermediate link in various other areas involving dual energy sources such as battery charging.

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