

DESIGN OF FPAA PROTOTYPE FOR PRACTICAL STUDYING OF MODIFIED VAN DER POL EQUATION

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The paper proposes and investigates FPAA prototype of sinusoidal oscillator based on the modified Van der Pol equation. Formulas for determination of basic parameters of the oscillations are presented. The proposed circuit is implemented and investigated by using AN221E04 Evaluation board of Anadigm Inc. The obtained experimental results demonstrate the basic theoretical relations and the possibilities for control of frequency and amplitude of the signal by using appropriate configuration and programming of FPAA blocks of the circuit.

Keywords: Field Programmable Analog Array, FPAA, Van der Pol equation, FPAA prototyping, sinusoidal oscillator

1. INTRODUCTION

The Van der Pol equation is a theoretical basis of great number signal generators. This is the reason for its extensive research. For instance, paper [1] presents careful study of generalised differential Van der Pol equation as applying Melnikov theory.

One of the basic approaches for studying of differential equations is computer modelling. The paper [2] present profound analysis and Simulink modelling of modified Van der Pol equation in order to provoke sinusoidal signals oscillation. The obtained results are good reason to extend the investigations by designing and examining a real prototype that models the discussed equation. Nowadays the most widely used platforms for practical prototyping and examination of analog and mixed-signal circuits are the FPAAs of Anadigm Inc. They allow easy implementation and quick reconfiguration of the examined prototype [3].

The goal of the paper is to propose and to investigate experimental FPAA prototype for modelling of modified Van der Pol equation.

2. FPAA MODELLING OF VAN DER POL EQUATION

Fig.1 shows the proposed FPAA prototype for modelling of modified Van der Pol equation. It is based on the improved Simulink model, presented in [2]. The circuit is implemented by using FPAA AN221E04 of Anadigm Inc. It consists of the following blocks: two integrators (Int.1 and Int.2) with possibility for programming of the integration constants k_1 and k_2 between $0.0025 [1/\mu S]$ and $3 [1/\mu S]$; two-input inverting sum stage SumInv; Multiplier; Hold circuit; Transfer Function block for

implementation of $\frac{1-cx^2}{b}$ function; and inverting gain stage GainInv with programmable gain G between 0.01 и 100. The lowest clock frequency of the circuit is $f_c = 250kHz$. The output sinusoidal signal appears at node A .

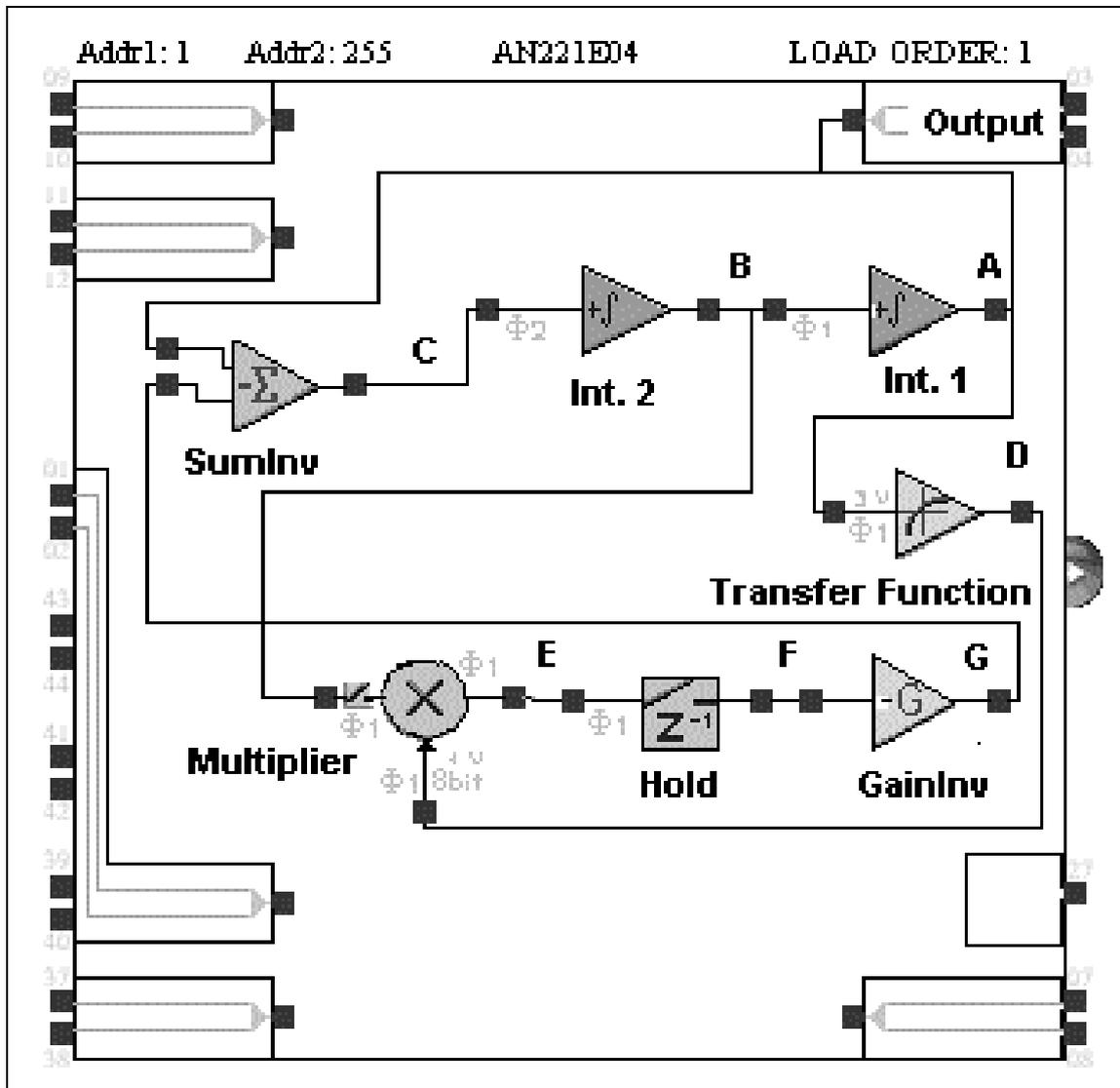


Fig.1. FPAA prototype for modelling of modified Van der Pol equation.

The analysis of the presented prototype determines the following expressions for the signals at different nodes of the circuit:

- node A (the output of Int.1) $\rightarrow x$;
- node B (the input of Int.1) $\rightarrow \frac{1}{k_1} \frac{dx}{dt}$;
- node C (the input of Int.2) $\rightarrow \frac{1}{k_1 k_2} \frac{d^2 x}{dt^2}$;
- node D (the output of Transfer Function block) $\rightarrow \frac{1-cx^2}{b}$;

- node **E** (the output of Multiplier) $\rightarrow \frac{1 - cx^2}{bk_1} \frac{dx}{dt}$;
- node **F** is identical with node **E** (the Hold circuit is included to joint the clocks of the Multiplier and inverting gain stage GainInv) $\rightarrow \frac{1 - cx^2}{bk_1} \frac{dx}{dt}$;
- node **G** (the output of the GainInv) $\rightarrow -\frac{G}{bk_1}(1 - cx^2) \frac{dx}{dt}$;
- node **C** (the output of the SumInv) $\rightarrow -x + \frac{G}{bk_1}(1 - cx^2) \frac{dx}{dt}$.

In the above terms, the signal at node **C** is defined in two forms - as a signal at the input of Int.2 and as a signal at the output of the SumInv. The equalization of the both terms leads to the modified Van der Pol equation for sinusoidal signals [2]:

$$(1) \quad \frac{1}{k_1 k_2} \frac{d^2 x}{dt^2} = -x + \frac{G}{bk_1} (1 - cx^2) \frac{dx}{dt},$$

$$(2) \quad \frac{d^2 x}{dt^2} - \frac{Gk_2}{b} (1 - cx^2) \frac{dx}{dt} + k_1 k_2 x = 0.$$

Based on the results in [2], the frequency f and the amplitude A_m of the FPAA sinusoidal oscillator are:

$$(3) \quad f = \frac{\omega_0}{2\pi} = \frac{\sqrt{k_1 k_2}}{2\pi},$$

$$(4) \quad A_m = \sqrt{\frac{4}{c}},$$

on condition that:

$$(5) \quad \left| \frac{\varepsilon}{\omega_0} \right| = \left| \frac{Gk_2}{b\sqrt{k_1 k_2}} \right| = \left| \frac{G}{b} \sqrt{\frac{k_2}{k_1}} \right| \ll 1.$$

The equations (3), (4) and (5) show the basic factors, which determinate the parameters of the oscillated signals. These factors are:

- Integration constants k_1 и k_2 , which determine the frequency of oscillations;
- The coefficient c that determines the amplitude of the signal;
- The gain G of the SumInv block - it should be sufficiently smaller in order to fulfil the condition (5).

3. EXPERIMENTAL RESULTS

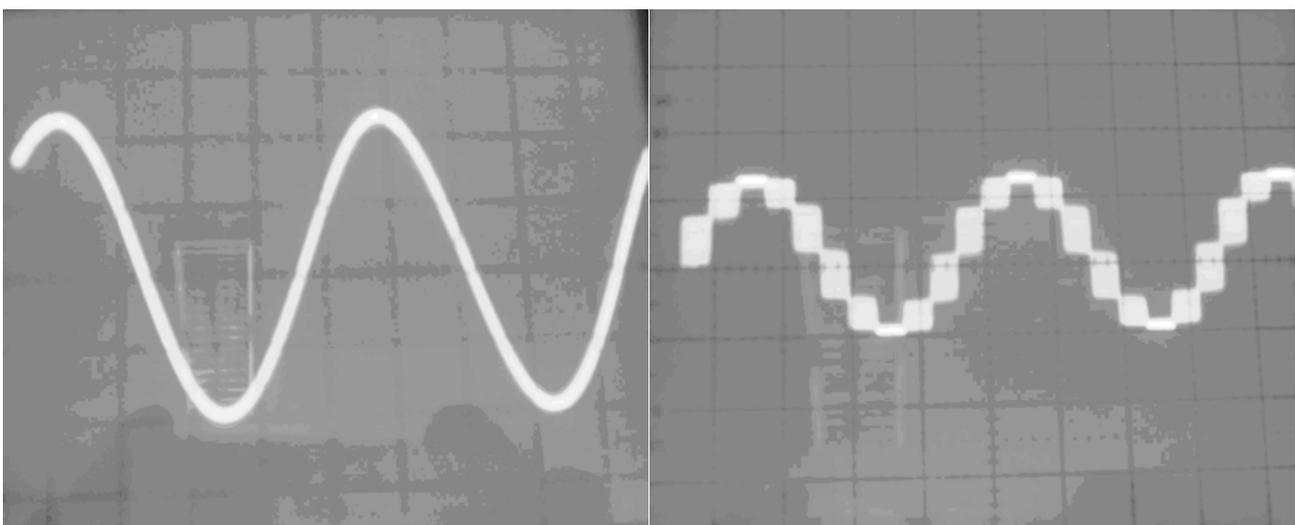
The proposed circuit was implemented and investigated by using AN221E04 Evaluation board of Anadigm Inc. Four types of experiments were carried out:

3.1. Investigation of amplitude and frequency of the signals in case of $k_1 = k_2$.

Tabl.1 shows the results from the investigation of amplitude and frequency of the output signal for different values of $k_1=k_2$. In these experiments, the coefficient $c=1$, so that expected amplitude of the signal is $A_m=2V$. The coefficient b equals 2.65, which ensures the limitation of the amplitude of the signal at the output of block Transfer Function between $\pm 3V$. The method for calculation and loading of the desired transfer function in FPAA is described in details in [3] and in [4]. The coefficient G is 0.1, which assures fulfilling of the condition (5). Third, fourth and fifth columns of the table contain: the frequency f_{calc} , calculated according to (3); the measured frequency f_{meas} and the relative error δ_f . Sixth and seventh columns contain the measured amplitude of the signal A_{meas} and its error δ_A toward the calculated value $A_m=2V$. The last column gives the ratio $n=f_c/f_{calc}$ between the frequency of clock signal of FPAA ($f_c=250kHz$) and the frequency of the oscillator f_{calc} . In this case f_c is the sampling frequency of SC circuits, which build the blocks of the prototype. The obtained results show very good coincidence for the frequency of the signal, but the limitation in the slew rate of the used OpAmps leads to high distortions in the form of the signal, when the ratio n decreases. This is shown on Fig.2. The best results are in the lower part of the range - in frequencies about several kilohertz (see rows 1÷5 of the Tabl.1).

Tabl.1. Results from investigation of the circuit in case of $k_1=k_2$; $G=0.1$; $c=1$; $b=2.65$; $A_m=2V$.

No	$k_1=k_2, 1/\mu S$	f_{calc}, Hz	f_{meas}, Hz	$\delta_f, \%$	A_{meas}, V	$\delta_A, \%$	n
1	0.0025	397.9	398.8	0.23	2.1	5	628.32
2	0.005	795.8	796.5	0.09	2.05	2.5	314.16
3	0.01	1591.5	1594.7	0.21	2.025	1.3	157.08
4	0.02	3183.1	3171.5	-0.36	1.95	-2.5	78.54
5	0.04	6366.2	6337.8	-0.45	1.85	-7.5	39.27
6	0.08	12732.4	12648.5	-0.66	1.55	-22.5	19.63
7	0.16	25465.8	25812.5	-1.37	1.15	-42.5	9.82



$n=628.32$

$n=9.82$

Fig.2. Form of the signals for two values of n .

3.2. Investigation of amplitude and frequency of the signals in case of $k_1 \neq k_2$.

Tabl.2 shows the results from investigation of values of frequency and amplitude of the output sinusoidal signal in case of $k_1 \neq k_2$. The coefficients $G = 0.1$; $c = 1$; $b = 2.65$ are the same as in the precedent experiments. The product of k_1 and k_2 is $k_1 \cdot k_2 = 0.0004$. Consequently, the expected values of the frequency and amplitude are $f_{calc} = 3183.1 \text{ Hz}$ and $A_m = 2V$. The presented results give the conclusion that the asymmetry in the values of the coefficients k_1 and k_2 has a weak influence over the frequency of the signal and strong influence over the amplitude. This is highly manifested in case of $k_2 \gg k_1$ (row 1 and row 2 of Tabl.2), when the condition (5) is not fulfilled.

Tabl.2 Experimental results in case of: $k_1 \neq k_2$; $G = 0.1$; $c = 1$; $b = 2.65$; $A_m = 2V$, $f_{calc} = 3183.1 \text{ Hz}$

No	$k_1, 1/\mu\text{S}$	$k_2, 1/\mu\text{S}$	$\left \frac{\varepsilon}{\omega_0} \right = \left \frac{G}{b} \sqrt{\frac{k_2}{k_1}} \right $	A_{meas}, V	$\delta_A, \%$	f_{meas}, Hz	$\delta_f, \%$
1	0.0025	0.16	0.302	0.6	-70	3057.7	-3.94
2	0.005	0.08	0.151	1.1	-45	3124.1	-1.85
3	0.01	0.04	0.075	1.95	-2.5	3170.3	-0.4
4	0.02	0.02	0.0375	1.95	-2.5	3171.7	-0.36
5	0.04	0.01	0.0185	2	0	3183.4	0.01
6	0.08	0.005	0.0094	2.2	10	3184.7	0.05
7	0.16	0.0025	0.0047	2.5	25	3187.9	0.15

3.3. Experiments for investigation of influence of coefficient G over the form of the output signal.

Tabl.3 shows the results from investigation of output signal of the circuit depending on the value of the gain of GainInv stage. This value determines directly the fulfilling of condition (5). The presented results shows that the frequency and the amplitude of the signal are relatively stable for these values of the gain G that are not higher than 1. Over this value, the condition (5) is not fulfilled and the form of the output signal distorts up to rectangular (row 11).

Tabl.3. Results from experimental investigation of dependency of the amplitude and the frequency of the signal from the value of G in case of $k_1 = k_2 = 0.02 [1/\mu\text{S}]$; $f_{calc} = 3183.1 \text{ Hz}$; $b = 2.65$; $c = 1$; $A_m = 2V$

No	G	$\left \frac{\varepsilon}{\omega_0} \right = \left \frac{G}{b} \right $	A_{meas}, V	$\delta_A, \%$	f_{meas}, Hz	$\delta_f, \%$
1	0.01	0.00377	2.25	12.5	3178.8	-0.14
2	0.02	0.00755	2.1	5	3178.1	-0.16
3	0.05	0.01887	2	0	3176.2	-0.22
4	0.1	0.03774	1.95	2.5	3171.6	-0.36
5	0.2	0.07547	1.95	2.5	3162.9	-0.63
6	0.5	0.18868	1.95	2.5	3135.6	-1.49
8	1	0.37736	1.95	2.5	3087.7	-3.12
9	2	0.75472	sinusoidal signal with distortions higher than 10%			
10	4	1.50943	triangular signal			
11	10	3.77358	rectangular signal			

3.4. Experiments for investigation of influence of coefficient c over the amplitude of the signal.

Tabl.4 demonstrates the possibility for control of amplitude of output sinusoidal signals by appropriate change of the value of coefficient c in equation (4). Four values of c , which determine respectively four values of A_m , are chosen for fixed value of the frequency. The experimental results demonstrate the perfect possibilities for programming of amplitude of the oscillated signal in the range between 0.5V and 3V (columns 6 and 7).

Tabl.4. Results from the investigation of influence of the coefficient c over the amplitude and the frequency of the signal in case of $G = 0.1$; $k_1 = k_2 = 0.02 [1/\mu S]$; $f_{calc} = 3183.1 Hz$.

№	A_m, V	c	b	$\left \frac{\varepsilon}{\omega_0} \right = \left \frac{G}{b} \right $	A_{meas}, V	$\delta_A, \%$	f_{meas}, Hz	$\delta_f, \%$
1	3	$\frac{4}{9}$	3	0.1	3	0	3155.8	-0.86
2	2	1	2.56	0.039	1.95	2.5	3171.7	-0.36
3	1	4	11.58	0.008636	1.05	5	3178.7	-0.14
4	0.5	16	47.3	0.002114	0.6	20	3180.3	-0.09

4. CONCLUSIONS

The paper proposes FPAA prototype (Fig.1) for modelling of modified differential Van der Pol equation. Formulas (3), (4) and (5) for determination of the parameters of the oscillated sinusoidal signal are presented.

The proposed circuit is implemented and investigated by using AN221E04 Evaluation board of Anadigm Inc. The discussed results confirm practically the basic theoretical relations in the equation and the possibilities for control of frequency, amplitude and form of the signal, by using appropriate configuration and programming of the blocks of the circuit.

The obtained results demonstrate very good agreement between scientific theory and experiment. This is a motive, for the future, to extend practical researches and demonstrations over to other types differential equations.

5. REFERENCES

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