# INVESTIGATION OF THE LINEARITY CRITERION USED BY THE SUBTRACTION METHOD FOR REMOVING POWERLINE INTERFERENCE FROM ECG

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*The detection of linear segments in ECG signals is the most important phase of the subtraction method for removing powerline interference. This is done by a criterion for linearity using second differences in the sampled signal. The criterion is interpreted as mathematical estimate of the acceleration within the segment and is substituted by a non-recursive digital filter. The transfer functions of simple and complex linearity criteria have been synthesized. They have been used for analysis of existing solutions. Conclusions have been formulated as a result of numerous and thorough experiments and a qualitative estimate of the criteria efficiency are confirmed. Requirements for the equivalent filter have been formulated. It was found that the filter analyzing the longest section of the signal is closest to the complex criteria and can be independently applied.*

**Keywords:** Digital filtering, ECG filtering, Interference rejection.

## 1. INTRODUCTION

The subtraction method for removing powerline (PL) interference from ECG signals was developed by Bulgarian scientists 20 years ago. The main advantage of the method is the almost total preservation of the signal spectrum including the range around the rated PL frequency. The exclusive advantages of the method are a continuous challenge to research and extension of its area of implementation. The structure of the method consists of three main phases:

1. Each sample (containing signal and interference) is checked for belonging to some linear section of ECG. Normally this is the PQ and/or TP interval but high and large T-wave may also include linear fragments.

2. The PL interference is removed by non-recursive symmetric digital filter type moving averaging preserving the useful signal only. Simultaneously, a subtraction between non-filtered and filtered signals results in current interference value, which is stored in a temporary buffer.

3. When the current sample does not belong to a linear segment, a value with the same interference phase is taken from the buffer to be subtracted from the current sample.

The first phase is of great importance for the subtraction method efficiency. Since there are no ideal linear sections within the ECG signals, a realistic criterion *Cr* for linearity had to be introduced. Many different types of criteria have been proposed and implemented in the course of time [3, 9]. However, their influence on the accuracy is still implicitly taken in consideration. Nevertheless, one may affirm that the most popular criteria use 'second differences' of the sampled signal.

## 2. THEORETIC STYDY OF THE LINEAR CRITERION

From mathematical point of view, the linearity (curvature) of a function is estimated by its second derivative (signal acceleration). In discrete alternative this is the 'second differences' *D*, obtained by subtraction of two 'first differences' *FD*. The first difference is derived from amplitudes of two samples and is a mathematical expression of signal velocity. When the first difference is taken from samples being located each other at distance of one or more periods of the PL frequency, the influence of the PL frequency on the linearity estimation is annulated.

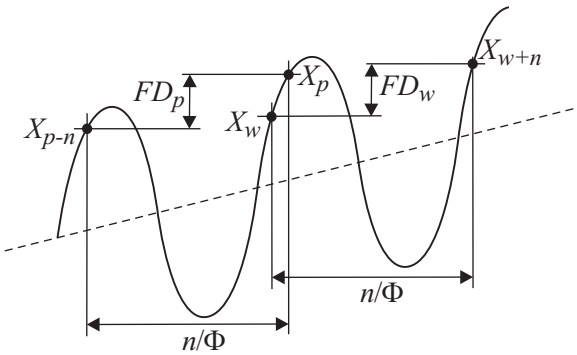


Fig.1. Contaminated samples located at one period of the PL frequency.

A linear segment superimposed by PL interference is shown in fig.1. The symbols used are: $F$ – frequency of the PL; $\Phi$ – sampling rate; $n = \Phi / F$ – number of samples within the interference period. It can be seen that the first differences covering whole periods are identical and the mean acceleration of the signal in the section is 0. This is true also for the second difference $D_i$ that means that its absolute value is convenient as a linearity criterion.

Actually the condition $Cr = |D_i| = 0$ is incorrect for real signals, therefore $|D_i| \le M$ is used, where $M$ is a criterion threshold (usually an experimentally found small constant value). The general equation for the linearity criterion related to a sample with time-coordinate $i$ is

$$D_{i,k} = (X_{i+n-k/2} - X_{i-k/2}) - (X_{i+k/2} - X_{i-n+k/2}), \quad k = 0, 1, \ldots, n-1. \qquad (1)$$

The parameter $k$ determines the length $2n-k$ of the investigated segment as well as the distance $n-k$ between the two first differences. Equation (1) can be interpreted as difference equation of a family of non-recursive symmetric filters, called for simplicity D-filters. Their amplitude response in the frequency domain is

$$D_{i,k}(f) = -4\sin\frac{n\pi f}{\Phi} \cdot \sin\frac{(n-k)\pi f}{\Phi}. \qquad (2)$$

For even $k$, the D-filters have linear phase response.

As mentioned above, different criteria have been used, e.g. $D_{i,4} = (X_{i+n-2} - X_{i-2}) - (X_{i+2} - X_{i-n+2})$ in [1], $D_{i,2} = (X_{i+n-1} - X_{i-1}) - (X_{i+1} - X_{i-n+1})$ in [2], all of them for $n = 5$. We found that in some specific cases, those linearity criteria 'slides' and detect pseudo-linear segments. This is illustrated in fig.2 by simulating an abrupt rise in linear signal with interference. The improper operation happens when the abrupt jump falls between points *w* and *p* on the signal. The two first differences become equal and the acceleration is wrongly recognized to be zero. Despite that in real ECG signals abrupt transitions would be sporadic, the criterion should be used carefully especially for a large distance between the points *w* and *p*.

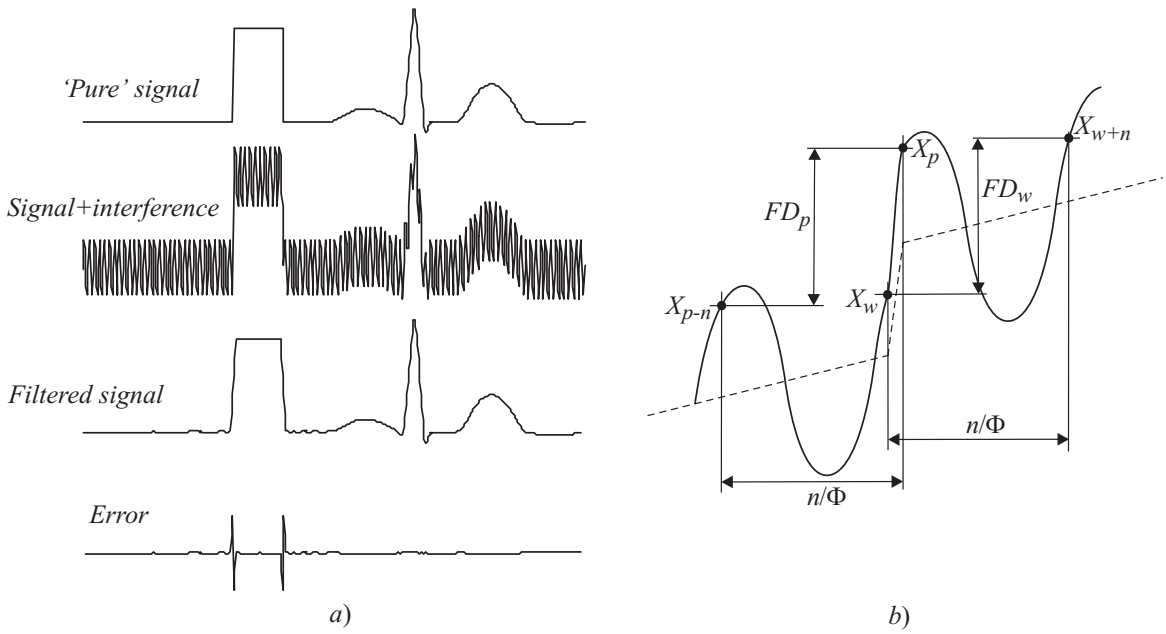


Fig.2. Example of incorrect operation of the linearity criterion in some specific cases.

## 3. ANALYSIS OF COMPLEX CRITERIA FOR LINEARITY

Complex criteria for linearity are mainly used with the subtraction method. The authors of one of the first papers on the problem [2] reported for $n$+1 successive first subtractions and a complex second subtraction presented as difference between the maximal and minimal first subtractions, which could be expressed as

$$Cr = \max\left(|D_0|, |D_1|, \ldots, |D_{n-1}|\right) < M \ . \qquad (3)$$

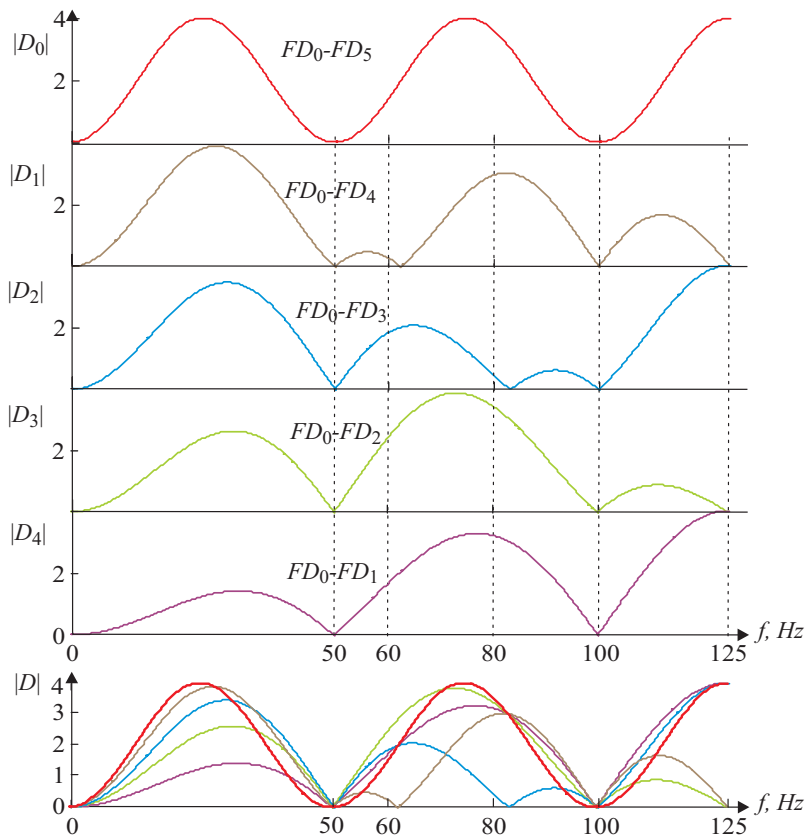


Fig.3. Complex linearity criterion for $\Phi$ = 250 *Hz*, $F$ = 50 *Hz*.

The amplitude responces of the family D-filters in case of $\Phi$ = 250 *Hz* and $F$ = 50 *Hz* ($n$=5) are shown in fig. 3. They are obtained in Matlab environment by entering the coefficient-vectors. The last graphic shows the summary (simultaneous) operation of all linearity criteria. The envelope of the family of curves forms the transfer function of the filter according the complex criterion (3). The D-filter $|D_0|$ ($k$=0), that analyzes the longest segment of the signal, is most closely to the complex one. ***$D_0$ is one and***

***the same for odd and even n*** and its trigonometric expression is

$$D_0(f) = -4\sin^2\frac{n\pi f}{\Phi}. \qquad (4)$$

***This filter does not detect pseudo-linearity and operates correctly***.

Another complex linearity criterion is used in [4]. The successive first differences $FD_0$ and $FD_1$ are taken in consideration. A segment of the signal is assigned as linear if the condition $|FD_1 - FD_0| < M$ remains consecutively $n$ times true. The criterion is given by:

$$Cr = \bigcap_{j=0}^{n-1}\left(\left|FD_{j+1} - FD_j\right| < M\right) = \bigcap_{j=0}^{n-1}\left(\left|D_{j,n-1}\right| < M\right). \qquad (4)$$

Later, the subtractions $FD_0$ and $FD_2$ are used [6]:

$$Cr = \bigcap_{j=0}^{n-1}\left(\left|FD_{j+2} - FD_j\right| < M\right) = \bigcap_{j=0}^{n-1}\left(\left|D_{j,n-2}\right| < M\right). \qquad (5)$$

Table.1. Coefficients of the second difference $D_7$ according to the complex criterion (4).

| Iteration | Coefficients | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 1 | | | | | | | |
| 2 | | 1 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 1 | | | | | | |
| 3 | | | 1 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 1 | | | | | |
| 4 | | | | 1 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 1 | | | | |
| 5 | | | | | 1 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 1 | | | |
| 6 | | | | | | 1 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 1 | | |
| 7 | | | | | | | 1 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 1 | |
| 8 | | | | | | | | 1 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 1 |
| Total | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |

The analysis of the complex criteria (4) and (5) is done in case of $\Phi$ = 400 *Hz* and F = 50 *Hz* ($n$=8). The coefficients of the second difference $D_7$ generated by the MATLAB function `freqz()`according to the criterion (4), are given in the first row of table 1. The criterion is satisfied 8 times successively that is equivalent to one-step right shift of the coefficients (shown in the next rows of the table). The last row gives the total effect of the complex criterion and the coefficients of the equivalent D-filter.

Analogously, the vector of the equivalent D-filter is obtained from the vector of the filter coefficients $\overrightarrow{\mathbf{D_6}} = [1\,0\,-1\,0\,0\,0\,0\,0\,-1\,0\,1]$ by means of formula (5) $\vec{\mathbf{D}} = [1\,1\,0\,0\,0\,0\,0\,0\,0\,-2\;\;-2\,0\,0\,0\,0\,0\,0\,1\,1]$. ***k+1 successive fulfillments of the D-filter*** $|\boldsymbol{D_k}|$, $k$ = 0, 1, 2, …, $n$-1, that lead to $Cr = |\boldsymbol{D_0}|$**,** are quite enough for rejection of pseudo-linear segments using this type of complex criteria. The transfer functions of the D-filters are shown in fig. 4. It can be observed that the complex criterion (4) is equivalent to the second subtraction $D_0$ but the complex criterion (5) equals to twofold consecutive appliance of $D_0$, having the trigonometric expression

$$D(f) = -8\sin^2\frac{n\pi f}{\Phi}\cos\frac{\pi f}{\Phi}. \qquad (6)$$

The complex criterion for linearity reported in [6] amplifies to a greater extent the ECG frequencies less than 50 *Hz*, which is a ***logical confirmation of the decreased threshold M***, compared to that used in [4].

The analysis shows that all D-filters have zero transfer coefficients for the PL frequency *F* and its harmonics, as well as for the zero frequency that allows their removing from the linearity estimation. The first difference of the D-filters transfer function for *f* = 0 *Hz* ***is zero*** (the tangent in this point is a horizontal line) ***that ignores the slope of the linear segment from the estimate***.

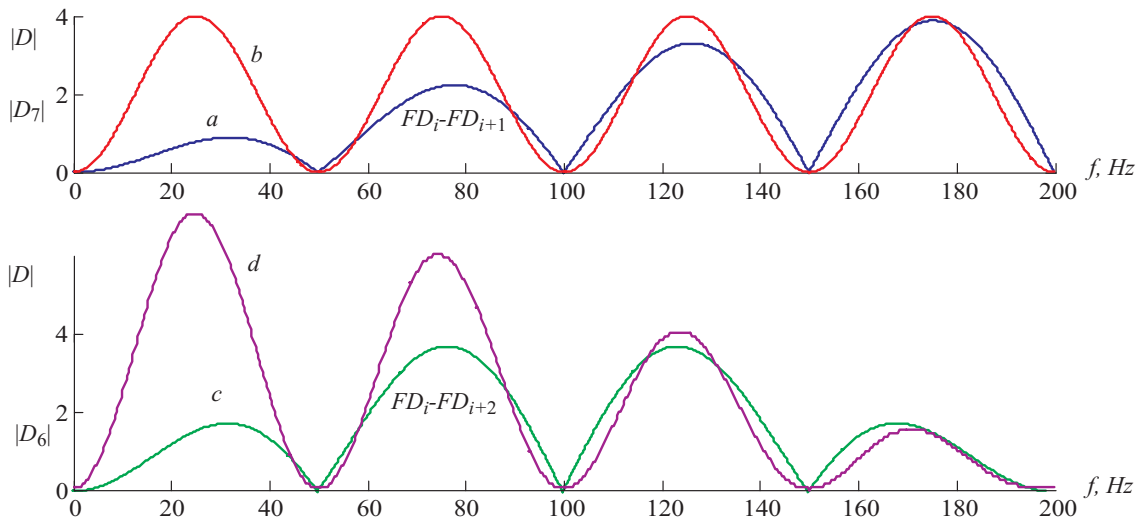


Fig.4. Transfer functions of D-filters: *a* – second difference $D_7$; *b* – complex criterion (4); *c* – second difference $D_6$; *d* – complex criterion (5).

Some of the simplest D-filters have transfer coefficient 0 also for other frequencies outside the interfering ones. Therefore, ECG segments with such frequency components can be recognized as pseudo-linear, thus extra PL interference will be read. Such criteria should be combined with convenient filters for extracting the interference frequencies only.

## 4. ANALYSIS OF THE LINEARITY CRITERION IN PRESENCE OF NOISES

The search for linear segment is greatly embarrassed in presence of EMG interference. An approach to linearity detection over moving averaged ECG signal in a parallel buffer is proposed in [8]. The transfer function of this linear criterion in the time domain $D^Y(f)$ is equivalent to the product of the transfer functions of the primary D-filter and K-filter for the averaged signal $D^Y(f) = D(f)K(f)$.

The moving averaged signal in the parallel buffer cannot contain abrupt transitions (like these simulated in fig. 1), therefore the shortest linearity criterion $D^Y_{i,n-1} = (Y_{i+(n+1)/2} - Y_{i-(n-1)/2}) - (Y_{i+(n-1)/2} - Y_{i-(n+1)/2})$ can be applied without risk of

detecting pseudo-linear sections. For a signal, filtered by $Y_i = \frac{1}{n}\sum_{j=-(n-1)/2}^{(n-1)/2} X_{i+j}$, $n$ being odd, a D-filter $D_{i,n-1}^{Y} = \left(X_{i-n} - 2X_i - X_{i+n}\right)\frac{1}{n} = D_{i,0}/n$ is obtained for the original signal. The criterion efficiency is like that of $Cr = |D_0| < nM$, therefore one may conclude that ***applying a criterion for linearity to a preliminarily filtered signal is equivalent to applying a criterion with increased threshold M to a non-filtered signal***. Analogous results are obtained with the other criteria too.

A dynamical control of the threshold *M* according to the PL interference amplitude is proposed in [5]. The necessity of increased threshold *M* in presence of EMG interference is reported in [7], where the dynamic *M* variation follows a noise/signal ratio currently calculated within approximately one RR interval. An applicable in real time advanced solution, using special 'wings' functions is given in [10].

## 5. CONCLUSIONS

The second difference, participating in linearity detection is analyzed as a ***non-recursive digital filter***, called D-filter. The obtained transfer functions are used for efficiency assessment of already published linear criteria. ***The following requirements towards the equivalent D-filter of the linearity criterion*** are formulated: zero transfer coefficient at the PL frequency; zero transfer coefficient at the frequencies ***close*** to zero. We found that the D-filter $|D_0|$, which analyses the longest section of the signal is closest to the complex criteria and can be used independently as a criterion for linearity.

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