Design Principles for Low-Noise Harmonic Second-Order Oscillators

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Abstract

In a previous paper [1] methodologies were presented for the design of high-performance relaxation oscillators. In this paper, methodologies are presented for the design of low-noise second-order oscillators. After an introduction to the world of second-order oscillators, basic resonator theory is put under a microscope. Optimization of the behavior of second-order oscillators is discussed on the system level and the interface between amplifier and resonator is optimized. A new view on resonator tapping is presented, including ideal undamp circuits for tapped resonators. After the theory of second-order oscillators, a comparison with the features of first-order oscillators shows the complimentarity of their merits. Therefore, this paper concludes with a discussion on systems with orthogonalized design criteria.

1 Introduction to second-order oscillators

As we did for first-order oscillators in [1], also for second-order oscillators we can find several basic functions in second-order oscillators. In these oscillators always a filter is present to give the oscillator its *frequency of preference*. The losses in this filter are compensated for by an undamping circuit, consisting of an amplifier and a control loop to define the amplitude of the oscillation. This control loop can either be linear or non-linear and either have a dynamical or a memoryless (instantaneous) behavior. For example, a second-order oscillator with an Automatic Gain Control (AGC) to adjust the level of undamping of the undamping circuit would yield a dynamical system that can be considered linear after settling. In this paper we will consider only static systems with some form of non-linearity to define the amplitude of the oscillation, as these systems are simple, can be designed accurately and yield a high performance [2]. Summarizing, in a second-order oscillator we find the following basic functions:

- Filtering; for frequency selectivity.

- Amplification; to supply the losses, either linear or non-linear.

- Amplitude control; to define the amplitude, either instantaneously or dynamically.

In this paper we first take a look at the filtering function, by taking a close look at the resonator. In section 3.1 we look at the amplification function and the influence of non-linearity in this function. After that the interface between the filter and the amplification function is optimized.

2 Resonator basics

As the resonator is both the source and the load of the active part of the oscillator, it is important to know its properties before designing the active circuitry. In low-noise, low-voltage circuits the series resonator is preferred because of its low impedance at resonance. A low-noise circuit can only be achieved when there is a relatively high power in the resonator, which implies the use of low impedances in low-voltage circuits. A practical LRC-resonator (including a parasitic C in parallel) has an impedance characteristic like the one depicted in figure 1.   Important aspects of
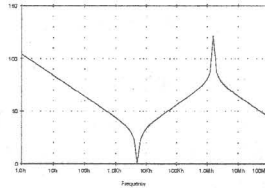

Figure 1: The impedance characteristic of a practical series resonator.

a resonator are of course its resonance frequency $\omega_0 = \frac{1}{\sqrt{LC}}$, where the impedance is purely resistive and at its minimum, and the quality factor Q, which is defined as:

$$Q = \frac{\omega_0}{2} \left| \frac{d\phi}{d\omega} \right|_{\omega_0} \tag{1}$$

When the quality factor of the resonator is high, it has a steep frequency-to-phase characteristic and noise induced phase varations in the oscillator loop can only have a small influence on the oscillation frequency, so a high frequency stability results. Strictly theoretically spoken, the optimal oscillation frequency is not the frequency at which the quality factor is at
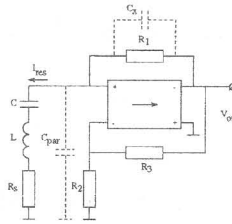
Figure 2: A basic topology of a second-order oscillator.

its maximum, but the frequency at which the frequency-to-phase slope is at its maximum. These frequencies are however almost the same in the case of a high Q resonator and even in the case that an extremely low Q resonator with a quality factor of 1 is used, the maximum improvement that can be reached by operation at this 'optimal' frequency is limited to about .65dB. Further, it would require a very accurate phase-shift in the active circuitry to make the oscillator operate at this frequency, so this understanding is nice in theory, but of no practical meaning whatsoever. Undamping a series resonator ideally can thus be done by the nullor configuration of figure 2, which forms a negative resistance of $-R_{in} = -R_1 \frac{R_2}{R_3} = -R_s$ at its input, and amplifies the resonator current to a voltage at the output. In practice always a parasitic capacitance is present at the input of the amplifier. When this is not taken into account when undamping the resonator, this parasitic capacitance together with the negative resistance of the oscillator forms a pole in the right half plane and parasitic oscillation at a higher frequency results. This problem can be overcome by canceling the parasitic capacitance by placing a negative capacitance in parallel to it. This can be accomplished very easily by placing a capacitance in parallel to, for example R_1.

3 Optimization of second-order oscillators

3.1 Optimization of the system; linear or non-linear undamping circuits.

In this section we take a look at the system of figure 3. In this system the resonator is undamped by a time-invariant non-linear amplifier [2]. Ideally, the amplification of the amplifier would be such that the loop gain of this system is exactly equal to 1. As it is impossible to implement a gain of exactly 1, the small-signal gain of the amplifier is such that the loop gain
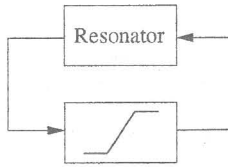
Figure 3: A simple model for a negative-feedback oscillator.

is larger than 1, but due to the limiting characteristics of the amplifier, the system is able to reach a steady state. The higher the small-signal gain of the limiting amplifier, the more time it will spend in the non-linear region. This non-linearity will cause 'folding' of noise from higher frequencies to the resonance frequency, which deteriorates the noise performance of the system. With the small-signal gain of the amplifier equal to A and the transfer of the resonator at the resonance frequency equal to H_0 we define the excess loop gain of the circuit as AH_0. For excess loop gains equal to or larger than 2, the phase-related output spectrum of the system depicted in figure 3 can be calculated to be:

$$S_{no}\phi(f_0 + \Delta f) = \left[\frac{1}{2Q\Delta f/f_0}\right]^2 \frac{1}{2}AH_0 \ S_{ni}(f) \tag{2}$$

This result shows that the phase-related output power is a factor AH_0 larger than the phase-related output in a linear oscillator [2]. Thus we can conclude that using an excess loop gain of 2 greatly simplifies the design of the oscillator, as no AGC circuit is required, and it only doubles the phase noise power.

3.2 Optimization of the interface between resonator and un-damp circuit; tapping.

Due to the very low impedance of series resonators at resonance and the very high impedance of parallel resonators, a noise match between the resonator and the undamping circuit is often difficult to obtain. When a series resonator is used, the impedance is so low that very often the equivalent voltage noise source at the input of the undamping circuit will dominate the noise behavior. It would be ideal to use an impedance transformer to obtain a noise match. Then a better trade-off between the noise voltage and the noise current could be made, yielding a better overall performance.

However, real transformers are far from ideal and it is very difficult to integrate them in IC's. To overcome this problem, tapping is used to get an impedance transformation at one specific frequency. In figure 4A a ca-
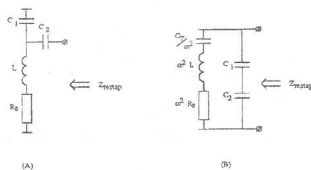

Figure 4: A capacitively tapped series resonator and the equivalent circuit.

pacitively tapped series resonator is depicted. Due to the tap, the output *voltage* at the resonator terminals increases and the output *current* decreases. This implies that compared to the larger output voltage, the noise voltage of the undamping circuit becomes less significant. The price that is paid for this is an increase of the influence of the noise current. This put a limit on the amount of tapping; finally the increase of the contribution of the noise currents overwhelms the decrease of the noise voltages. In the circuit of figure 4A a tapping factor can be defined as:

$$\alpha = 1 + \frac{C_1}{C_2} \tag{3}$$

Using this tapping factor, an equivalent resonator can be found as shown in figure 4B. The tapped resonator can be seen as a series resonator with new values for L, R and C in parallel with a capacitance comprising the two capacitors involved in series connection (C_T is the parallel equivalent of both capacitors). When using tapped resonators care has to be taken not to go to far. When the tapping ratio is increased, also the signal coming from the resonator is decreased, so an optimal tapping ratio exists, depending on the resonator impedance, and the parameters of the transistors used in the negative-impedance circuit. For an example of such an optimization, where a 20dB better noise performance is achieved by the use of tapping, the reader is referred to [3, 4]. In the case of a tapped resonator, always an equivalent capacitance is present across its terminals, like the parasitic capacitance in the case of a normal series resonator. Therefore, a good undamping circuit would look like the one depicted in figure 2

3.3 Degradation of Q due to tapping

In the previous section it has been shown that tapping a resonator to transform the impedance at resonance can be favorable for the noise performance of an oscillator. However, tapping the resonator can affect its quality factor. This would counteract the improvement in the noise performance that was established by changing the impedance at resonance. To explain this we will again take a look at the equivalent of the tapped series resonator depicted in figure 4b. Due to the extra parallel capacitance, the tapped resonator also exhibits a parallel resonance, that comes closer to the series resonance as the tapping ratio is increased. In figure 5 we see that when the resonance modes are very close to each other, the phase slopes start to influence each other lowering the effective Q of the circuit, thus degrading the phase noise performance of the circuit.
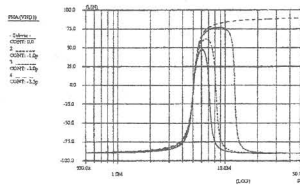

Figure 5: The influence of the frequency difference between parallel and series resonance on the phase at series resonance.

4 Coupled Second-Order Oscillators

In 1986, Driscoll [5] reported the development of a coupled system of two harmonical oscillators. In this coupled system, the SNR was improved 5dB for carrier offset frequencies $f \leq 1kHz$. In the design treated by Driscoll, care was taken to correlate the carriers, while noise sources far from the carrier remained uncorrelated. When the ouput signals of the oscillators are summed, the correlated summing of the carriers and the uncorrelated summing of the noise yields a 5dB improved SNR. To achieve this improvement, a good matching of the resonators is obligatory. When the resonance frequencies of these resonators are not exactly equal, the resulting average frequency of the compete oscillator system differs from the separate resonance frequencies and thus, neither of the two is running at its maximal Q. Another example of the use of coupled resonators is given in [6]. In this

paper a monolithic LC VCO is proposed, which uses two LC resonators to achieve a tuning range of 200MHz at 1.8GHz. The resulting oscillation frequency is a weighted average of the resonance frequencies of the two resonators. When the oscillation frequency of the oscillator is equal to the resonance frequency of one of the resonators, the phase noise performance of the oscillator is determined by the maximal Q of this resonator. However, when the oscillator is somewhere in the middle of the tuning range, none of the resonators have much selectivity, yielding a very bad phase slope and an extremely bad noise performance. In this case, also a relaxation oscillator could have been used [1]. A last example of coupled resonators is given in [7]. In this paper, a coupled resonator oscillator is used as a local quadrature oscillator. Also in this oscillator, two resonators determine the frequency of the oscillation and as they are not exactly matched, the resulting Q of the oscillator will always be lower than that it would have been when one resonator had been used. Also, mismatch between the resonators will give an error in the quadrature relation, yielding a suboptimal performance. When good quadrature oscillators are required, it is better to take two coupled relaxation oscillators, as they have no frequency of preference [1, 8].

5 Orthogonalization

In this and the previous paper [1], we have seen that classification of oscillators is of crucial importance to choose the right oscillator for the job. Further, we have seen that the good features of the classes of first-order oscillators and the second-order resonator oscillators are rather complementary. First-order oscillators have good tuning characteristics, but are very noisy, while second-order oscillators can not be tuned easily, but have superior noise performance. This complementarity of the oscillators can be used to build systems making use of the good features of both types of oscillators. By orthogonalization of requirements, a better overall system performance can be obtained. In figure 6, a multiloop synthesizer is depicted as an example. When the rightmost loop is a wide-band PLL, the phase-noise of the high-frequency VCO is determined by the multiplied phase-noise of the reference oscillator in the leftmost loop. When the frequency divider of the rightmost loop is adjusted for tuning purposes, the required tuning range of the reference oscillator is reduced. Thus, for the reference VCO the tuning-range is reduced, while a good phase-noise performance is required. A second-order oscillator with a relatively high
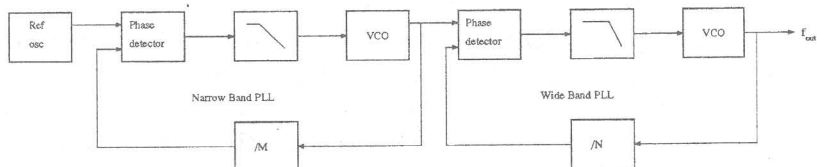
Figure 6: A multi-loop frequency synthesizer.

resonator power can meet these requirements. The VCO in the rightmost loop has to have the full tuning range, but the phase-noise performance is less important, because the phase-noise is reduced by the PLL suppression in the PLL bandwidth. A first-order oscillator can thus be used in this loop. When the output signal of the synthesizer also has to have an excellent long-term stability, this can be in conflict with the design requirements for the low-phase noise reference oscillator. The relatively high resonator power causes wear-out of the resonator and aging becomes important. Therefore, another loop is used to lock this oscillator to a resonator oscillator with low resonator power. When this loop has a small bandwidth, the phase noise performance of the reference signal is still determined by the high-power resonator oscillator. In this way, the specifications for the signal can be orthogonalized and each oscillator in the system can be optimized for only one quality aspect.

6   Conclusions

Second-order oscillators have shown to be oscillators with a frequency of preference and therefore they have an inherently better frequency stability than first-order oscillators. It has been shown that 'interface' problems exist between the actual resonator and the active circuit. Special techniques to connect both parts, like tapping, have been shown to be advantageous for the noise behavior when properly applied. It has also been shown that coupling two or more second-order oscillators to obtain a better frequency stability is very difficult and results in many cases in a degraded instead of an improved behavior. When the requirements on an oscillator signal are extremely stringent, orthogonalization can help us to build systems in which each feature of the signal is determined by one single oscillator, that can be optimized for that specific signal requirement. This yields a better overall performance than taking one oscillator that has to be able to meet all requirements.

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